DSGRN Group Meeting Presentation 3

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Problem

Take an interaction function $f(z_1, \ldots, z_n)$ and let

$$P_m := \{f(z_1, \ldots, z_n) : z_i = \ell_i, \ell_i + \delta_i^{(1)}, \ldots, \text{ or } \ell_i + \delta_i^{(1)} + \cdots + \delta_i^{(m-1)}, \forall i\}$$

$$P_m \subset \mathbb{R}[\ell_1, \ldots, \ell_n, \delta_1^{(1)}, \ldots, \delta_n^{(1)}, \ldots, \delta_1^{(m-1)}, \ldots, \delta_n^{(m-1)}] \text{ (}nm\text{ variables)}$$

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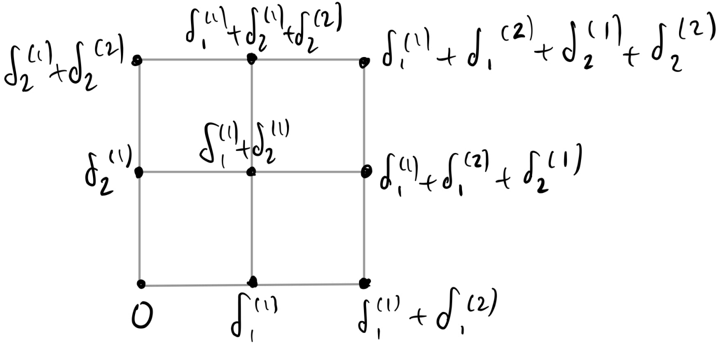
We want to find $T(P_m, \prec_m, (0, \infty)^{mn})$, which is all the realizable orderings of polynomials in P_m.
i.e We want to find the orderings for which there exists some positive values for $\ell_1, \ldots, \ell_n, \delta_1^{(1)}, \ldots, \delta_n^{(1)}, \ldots, \delta_1^{(m-1)}$ which places the polynomials into that order when evaluated.

Example of lattice being extended

$$\boxed{f = z_1 + z_2}$$

$$z_1 = \ell_1,\ \ell_1 + \delta_1^{(1)},\ \text{or}\ \ell_1 + \delta_1^{(1)} + \delta_1^{(2)}$$

$$z_2 = \ell_2,\ \ell_2 + \delta_2^{(1)},\ \text{or}\ \ell_2 + \delta_2^{(1)} + \delta_2^{(2)}$$



Isomorphic to the lattice of points on a subset of the coordinate plane ordered component-wise.

Results so far

At least in the linear case, this problem can use the same algorithm as used before, but there could be the limiting factor of the number of solutions growing very quickly as m or n grows larger.

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Possible next step: Create code to handle the linear case and see how many solutions there can be.